MAN/HEP/2006/4

Perturbative and Nonperturbative Contributions to a Simple Model for Baryogenesis

K. R. S. Balaji[1,2,3,*] and Björn Garbrecht[4,†]

[1] *Dept. of Physics, McGill University,*
3600 University Street, Montreal QC, H3A 2T8, Canada

[2] *Physique des Particules, Université de Montréal,C.P. 6128,*
succ. centre-ville, Montréal, QC, Canada H3C 3J7

[3] *Département des Sciences de la Terre et de l'Atmosphère,*
Université du Québec à Montréal, C.P. 8888,
succ. centre-ville, Montréal, QC, Canada H3C 3P8

[4] *School of Physics & Astronomy, Oxford Road, Manchester M13 9PL, UK*

Single field baryogenesis, a scenario for Dirac leptogenesis sourced by a time-dependent scalar condensate, is studied. We compare the creation of the charge asymmetry by the perturbative decay of the condensate with the nonperturbative decay, a process of particle production commonly known in the context of inflation as preheating. The nonperturbative channel dominates when the coupling of the scalar field to leptons is sufficiently large.

1. INTRODUCTION

Models for baryogenesis tie together cosmology and particle physics [1]. The discovery of small neutrino masses [2] and their explanation *via* lepton-number violating Majorana masses and the see-saw mechanism strongly supports the leptogenesis mechanism [3]. An alternative to this scenario is to assume pure Dirac mass terms for the neutrinos, that is the absence of Majorana masses, and to induce an asymmetry between the left- and right-handed neutrinos, which is subsequently turned into baryon number through sphaleron transitions.

*balaji@hep.physics.mcgill.ca

†bjorn@hep.man.ac.uk

This idea is referred to as Dirac leptogenesis [4].

In many scenarios for baryogenesis the necessary C and CP violation occur simultaneously, induced by a matrix of Yukawa couplings. An exception is electroweak baryogenesis [5, 6], where in first place a CP-violating axial asymmetry is produced which is subsequently not erased *via* equilibration. Parity violation is then contributed in a second step due to sphaleron interactions.

Also Dirac leptogenesis relies on the transition of an axial asymmetry into baryons through the sphaleron process. The initial asymmetry is stored within Dirac neutrinos and does not get erased due to equilibration until electroweak symmetry breaking since the Yukawa coupling to the Standard Model Higss field is tiny. While the CP asymmetry can be provided through a matrix of Yukawa couplings and the out-of equilibrium decay of heavy scalar particles into Dirac neutrinos [4], it has been suggested that also a single Dirac mass term can source CP, provided it is time dependent. This mass term can arise due to Yukawa couplings of neutrinos to a rolling scalar field, and the resulting mechanism has been named single field baryogenesis [7].

Interpreting the scalar condensate oscillating around *zero* as a large amount of scalar quanta at zero momentum, the axial asymmetry can be generated due to the perturbative decay of these particles, as commonly assumed in scenarios for Affleck-Dine baryogenesis [8]. However particles can also be produced nonperturbatively, as first pointed out in Ref. [9], a process which is often referred to as preheating in the context of the decay of the inflaton [10]. Various aspects of preheating from the decay of flat directions are discussed in [11].

In parallel, in the coherent baryogenesis [12] scenario, the oscillating condensate leads directly and at tree-level to the production of a charge asymmetry during preheating when it couples to matter such that a time-dependent C and CP violating mass matrix arises. Consequently, in the case of a single time-dependent mass term a preheating process can lead to an axial asymmetry [13] and thereby source single field baryogenesis.

In the present analysis, we focus on the importance of non-perturbative contributions to the baryon asymmetry, and we choose the single field model due to its simplicity. We emphasise nonetheless that nonperturbative particle production may be of relevance for other scenarios, *e.g* Affleck-Dine baryogenesis.

2. PERTURBATIVELY SOURCED SINGLE FIELD BARYOGENESIS

Let us begin by considering a simple toy-model potential

$$V = \frac{\mu^2}{2}\left(|\phi_u|^2 + |\phi_d|^2\right) + \frac{m^2}{2}\left(\phi_u\phi_d + \phi_u^*\phi_d^*\right) + \lambda_\nu L\phi_u\bar{\nu}_R + \lambda_\nu \bar{L}\phi_u^*\nu_R \ . \qquad (1)$$

The fields ϕ_u and ϕ_d are scalar and are multiplets of the electroweak group $G_{EW} = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, $\phi_u = \left(\mathbf{2}, \frac{1}{2}\right)$, $\phi_d = \left(\mathbf{2}, -\frac{1}{2}\right)$, while $L = \left(\mathbf{2}, -\frac{1}{2}\right)$ with the components

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \ , \qquad (2)$$

and $\nu_R = (\mathbf{1}, 0)$ are Weyl fermions. The scalar mass eigenstates are then $\frac{1}{\sqrt{2}}\left(\Im[\phi_u] + \Im[\phi_d]\right)$ and $\frac{1}{\sqrt{2}}\left(-\Re[\phi_u] + \Re[\phi_d]\right)$, both with mass $\sqrt{\mu^2 - m^2}$, and eigenstates, $\frac{1}{\sqrt{2}}\left(-\Im[\phi_u] + \Im[\phi_d]\right)$ and $\frac{1}{\sqrt{2}}\left(\Re[\phi_u] + \Re[\phi_d]\right)$ with mass $\sqrt{\mu^2 + m^2}$. The inflationary Hubble rate is given by H_I, and we assume that both of these mass eigenvalues are slightly below this value. Therefore, at horizon exit the scalar fields get amplified up to a magnitude $\sim H_I$ and a random direction in $\mathrm{SU}(2)_L$-space. Inflationary expansion then leaves behind a homogeneous vacuum expectation value for the scalar fields in our patch of the Universe. This induces large neutrino masses, such that they initially do not thermalise. In turn, the potential for $\phi_{u,d}$ does not get altered by thermal corrections.

Since we are interested in neutrino production, in the following, we consider only the neutral components $\phi^0_{u,d}$. In particular, since the asymmetry is produced from ϕ^0_u, we take this to be the source field as in the single field baryogenesis scenario. Furthermore, we assume $m \ll \mu$.

Coherent oscillations begin at the time when the Hubble rate has decreased to the value μ, and the solution for ϕ^0_u can be approximated for small small μ/H by

$$\begin{aligned} \Re[\phi^0_u] &= \left[A_1^R \cos\left(\sqrt{\mu^2 - m^2 t}\right) + A_2^R \cos\left(\sqrt{\mu^2 + m^2 t}\right)\right] a^{-3/2}(t) \ , \\ \Im[\phi^0_u] &= \left[A_1^I \cos\left(\sqrt{\mu^2 - m^2 t}\right) + A_2^I \cos\left(\sqrt{\mu^2 + m^2 t}\right)\right] a^{-3/2}(t) \ , \end{aligned} \qquad (3)$$

where the values of $A^{R,I}_{1,2}$ are random initial values arising from inflation, as described above, and $a(t)$ denotes the scale factor of the Universe, t denotes comoving time. In order to keep the present discussion simple, we assume $A_1^R = A^R$, $A_2^R = 0$, $A_2^I = A^I$ and $A_1^I = 0$ in our patch of the Universe. Under these conditions, the charge density carried by the field ϕ^0_u is given by

$$Q_\phi = \frac{\mathrm{i}}{2}\left(\phi_u^{0*}\dot{\phi}_u^0 - \phi_u^0\dot{\phi}_u^{0*}\right) \approx a^{-3}\mu A^R A^I \sin\left(\frac{m^2}{\mu}t\right) , \tag{4}$$

where we expanded in μ/m and neglected time derivatives acting on the scale factor. According to the interaction term with the leptons in the potential (1), Q_ϕ is transferred to a charge asymmetry within the left handed neutrinos when ϕ_u^0 decays. Due to conservation of total lepton number, a precisely opposite amount of the asymmetry is stored within the right handed neutrinos. However, this asymmetry in the right-handed sector is not transferred into baryons by sphalerons due to the left-handed nature of interactions.

We assume that the Universe is radiation dominated when coherent oscillations commence and that this remains so until the scalar fields decay, such that they contribute only negligibly to the entropy density s. Just like Q_ϕ, s scales down as a^{-3}, such that we find for the asymmetry within left-handed neutrinos at the time t_Γ, when the scalar field decays,

$$\frac{n(\nu_L) - n(\bar{\nu}_L)}{s} = \alpha \frac{A^R A^I}{\mu^{1/2} m_{Pl}^{3/2}} \sin\left(\frac{m^2}{\mu}t_\Gamma\right) . \tag{5}$$

Here, we have used the relations $H = 1.66 g_*^{1/2} T^2/m_{Pl}$, $s = \frac{2\pi^2}{45} g_* T^3$ and have taken $H \approx \mu$. The number of relativistic degrees of freedom is denoted by g_*, such that α is a numerical constant of order *one* for realistic values of g_*.

3. NONPERTURBATIVE SOURCE

Following Ref. [13], we calculate the axial asymmetry induced by the nonperturbative decay of ϕ_u^0. We do so by solving numerically the conformally rescaled Dirac equation

$$\left[\mathrm{i}\partial\!\!\!/ - m_R + \mathrm{i}\gamma^5 m_I\right]\psi = 0 . \tag{6}$$

We take

$$\psi = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} , \tag{7}$$

such that by the potential (1), we have

$$m_R = a\lambda_\nu \Re[\phi_u^0], \quad m_I = a\lambda_\nu \Im[\phi_u^0] . \tag{8}$$

Furthermore, we introduce the conformal time η, which is related to comoving time as $dt = a d\eta$, and we take $\partial_0 = \partial_\eta$.

Let us introduce the positive and negative frequency mode functions, $u_h(\mathbf{k},\eta)$ and $v_h(\mathbf{k},\eta) = -i\gamma^2(u_h(\mathbf{k},\eta))^*$, respectively. They form a basis for the Dirac field,

$$\psi(x) = \int \frac{d^3k}{(2\pi)^3} \sum_h \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}} \left(u_h a_h(\mathbf{k}) + v_h b_h^\dagger(-\mathbf{k})\right), \quad u_h = \begin{pmatrix} L_h \\ R_h \end{pmatrix} \otimes \xi_h\,, \tag{9}$$

where ξ_h is the helicity two-eigenspinor, $\hat{h}\xi_h = h\xi_h$. The Dirac equation then decomposes into

$$\begin{aligned} \mathrm{i}\partial_\eta L_h - h|\mathbf{k}|L_h &= m_R R_h + \mathrm{i}m_I R_h\,, \\ \mathrm{i}\partial_\eta R_h + h|\mathbf{k}|R_h &= m_R L_h - \mathrm{i}m_I L_h\,. \end{aligned} \tag{10}$$

From L_h and R_h, we can define the quantities

$$\begin{aligned} f_{0h} &= |L_h|^2 + |R_h|^2, \qquad f_{3h} = |R_h|^2 - |L_h|^2, \\ f_{1h} &= -2\Re(L_h R_h^*), \qquad f_{2h} = 2\Im(L_h^* R_h), \end{aligned} \tag{11}$$

where f_{0h} is the charge density, f_{3h} the axial charge density, f_{1h} the scalar density and f_{2h} the pseudoscalar density. Note that one can easily show that f_{0h} is conserved by Eq. (10), reflecting the charge conservation of the Dirac neutrinos.

The initial conditions corresponding to a particle number $n_h(\mathbf{k}) = |\beta_0|^2$ are

$$\psi_\mathbf{k} = \begin{pmatrix} \alpha_0 L_h^+ + \beta_0 L_h^- \\ \alpha_0 R_h^+ + \beta_0 R_h^- \end{pmatrix}, \qquad |\alpha_0|^2 + |\beta_0|^2 = 1\,, \tag{12}$$

where

$$\begin{aligned} L_h^+ &= \sqrt{\frac{\omega(\mathbf{k}) + hk}{2\omega(\mathbf{k})}}, \qquad L_h^- = -i\frac{m}{|m|}\sqrt{\frac{\omega(\mathbf{k}) - hk}{2\omega(\mathbf{k})}}, \\ R_h^+ &= \frac{m^*}{\sqrt{2\omega(\mathbf{k})(\omega(\mathbf{k}) + hk)}}, \qquad R_h^- = i\frac{|m|}{\sqrt{2\omega(\mathbf{k})(\omega(\mathbf{k}) - hk)}}, \end{aligned} \tag{13}$$

and $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + |m|^2}$. Since we assume to have initially zero neutrinos, we take $\beta_0 = 0$ in the following.

When ϕ_u^0 ceases to oscillate, the particle number is given by

$$n_h(\mathbf{k}) = \frac{1}{2\omega(\mathbf{k})}\left(hkf_{3h} + m_R f_{1h} + m_I f_{2h}\right) + \frac{1}{2}\,. \tag{14}$$

Of course there is no charge asymmetry, since there is an opposite amount of antiparticles. However, when $m_I \neq 0$, an asymmetry in the number of particles with positive $(h = +)$ and negative $(h = -)$ helicity may be generated. Note that in the limit $m_R, m_I \to 0$, $n_{\mathbf{k}h} = \frac{1}{2}hf_{3h} + \frac{1}{2}$, since then chirality and helicity coincide. Therefore,

$$2\,(n_+ - n_-) = f_{3+} + f_{3-} \tag{15}$$

when the masses vanish. The factor *two* on the left hand side occurs because the total axial asymmetry gets contributions from particles and antiparticles, while $n_h(\mathbf{k})$ counts just the particles.

With a prime denoting a derivative *w.r.t.* η, the scalar equation of motion reads

$$\phi'' + 2\frac{a'}{a}\phi' + a^2\frac{dV}{d\phi} + a\Gamma\phi' = 0\,. \tag{16}$$

During radiation expansion, $a = a_R\eta$, and when $H = a'/a^2 \ll \sqrt{\mu^2 \pm m^2}$, the solution to this equation is well approximated by

$$\phi_u^0 \approx \left[A^R\cos\left(\sqrt{\mu^2 - m^2}\frac{a_R}{2}\eta^2\right) + \mathrm{i}A^I\cos\left(\sqrt{\mu^2 + m^2}\frac{a_R}{2}\eta^2\right)\right](a_R\eta)^{-3/2}\mathrm{e}^{-\frac{1}{4}\Gamma a_R\eta^2}\,, \tag{17}$$

with the same assumptions for the real and imaginary parts as in the previous section. We use this solution to obtain the Dirac neutrino mass term (8) and numerically solve Eq. (10) by integrating up to the time when $\Gamma > H$, such that the Dirac mass term ceases to oscillate and the axial charges $f_{3h}(\mathbf{k})$ get frozen in. A typical plot of the spectrum of the generated charge charge asymmetry is given in FIG. 1. Particle production occurs at a time t_{Res} when the fermionic mode is in resonance with the coherently oscillating field. The production of the soft modes with small momentum k is suppressed because the initial charge asymmetry in the scalar field is small $(m^2/\mu)t_{Res} \ll 1$, *cf.* Eq. (5). Consequently, the production of asymmetry within modes which resonate later becomes stronger first. Eventually, there is a damping effect due to the red-shifting of the oscillating condensate and finally due to its decay at the rate Γ. Note that due to Pauli blocking $-2 \leq f_{3+}(\mathbf{k}) + f_{3-}(\mathbf{k}) \leq 2$.

Of course, the axial asymmetry vanishes in the case when the scalar charge (4) is *zero*. When $A^I = 0$, the term $\propto \gamma^5$ in the Dirac equation (6) vanishes and there is obviously no CP-violation. When $A^I \neq 0$ but $m = 0$ there is also *zero* scalar charge. Since then the phase is constant, $\partial_\eta \mathrm{arg}(m_r + \mathrm{i}m_I) = 0$, the γ^5-term can in principle be removed at all times by a rephasing of the fermionic field. Consequently, even if we do not perform this rephasing, we expect to find $f_{3+}(\mathbf{k}) + f_{3-}(\mathbf{k}) \equiv 0$, which can also be verified numerically.

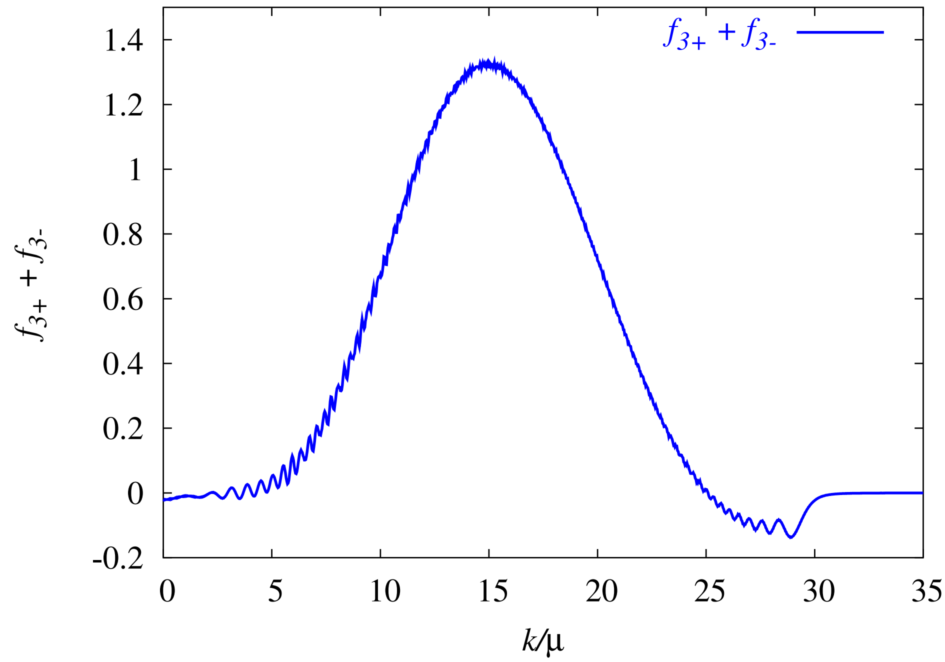

FIG. 1: The axial asymmetry plotted over momentum k, which is taken to be the physical momentum at the time when coherent oscillations begin. The choice of parameters is: $\lambda_\nu = 0.2$, $A^R = A^I = 20\mu$, $m = 0.05\mu$, $\Gamma = 0.002\mu$.

The axial charge density stored within the neutrinos is is the integral over the asymmetry within the modes

$$Q_A = \int \frac{d^3k}{(2\pi)^3} \left(f_{3+}(\mathbf{k}) + f_{3-}(\mathbf{k}) \right) , \tag{18}$$

where k is to be understood as the physical momentum at the time when coherent oscillations begin. The axial asymmetry to entropy ratio then turns out to be

$$\frac{n(\nu_L) - n(\bar{\nu}_L)}{s} = \alpha \frac{Q_A}{(\mu m_{Pl})^{3/2}} , \tag{19}$$

which we want to compare with the perturbative result (5).

We denote the axial densities $n(\nu_L) - n(\bar{\nu}_L)$ by ρ_{res} for the nonperturbative or resonant case of Eq. (18) and by ρ_{pert} for the perturbative decay as expressed in Eq. (5). The initial amplitudes of the scalar field are chosen to be $A^R = A^I$. We display the produced axial asymmetries over the initial amplitudes in FIGs. 2 and 3, where we have taken different values for the damping rate Γ. We find that depending on the value of Γ the exact value of the asymmetry could be larger or smaller in the nonpertubative case. This also follows from the analytic expression given in (17) for the neutral component of the field amplitude ϕ_u^0.

Let us first consider the case where the perturbative decay dominates.

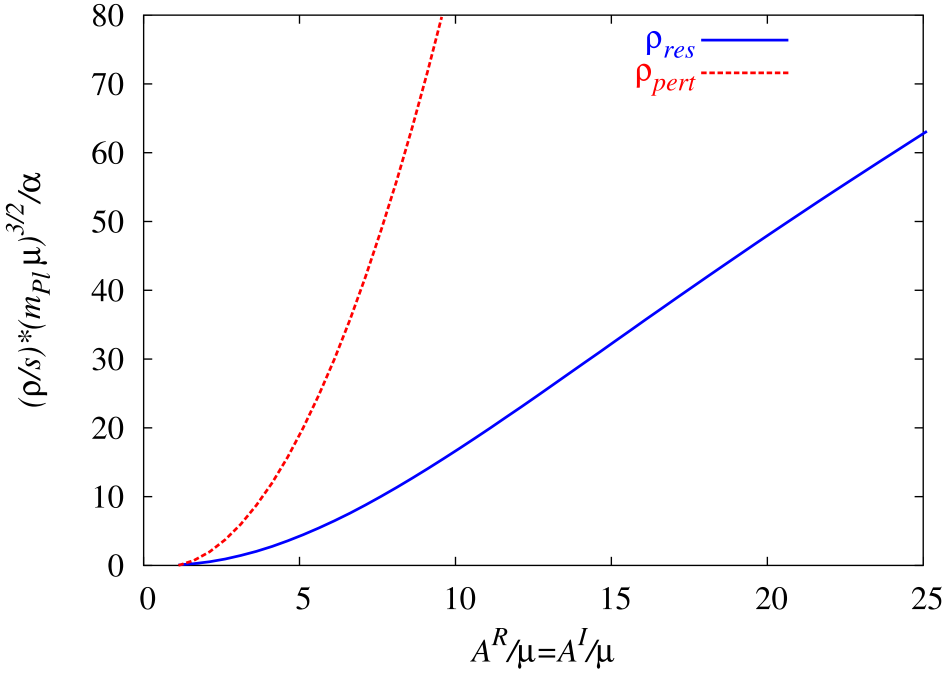

FIG. 2: The axial asymmetry plotted over the initial amplitude of $\phi_u^0 = A^R + \mathrm{i}A^I$. The choice of parameters is $\lambda_\nu = 0.2$, $m = 0.05\mu$, $\Gamma = 0.01\mu$.

This is shown in in figure 2, where perturbative production dominates the resonant source. However, when a larger value for the coupling λ_ν is chosen, ρ_{res} gets enhanced. Moreover, note that while initially ρ_{pert} and ρ_{res} grow as the square of the initial scalar amplitude, ρ_{res} gets suppressed for large amplitudes due to Pauli blocking, which we do not take into account in our formula for the perturbative asymmetry (5).

When we choose a smaller value for the damping rate Γ, the situation is as plotted in figure 3. The nonperturbative, resonant source dominates over the perturbative decay because coherent oscillations last longer and a larger phase space volume may be filled as the fermionic modes are red-shifted. In either case, we should observe that the total contribution is always larger than the individual contribution. The extent of the contribution (as shown here) is strongly dependent on the value of Γ chosen.

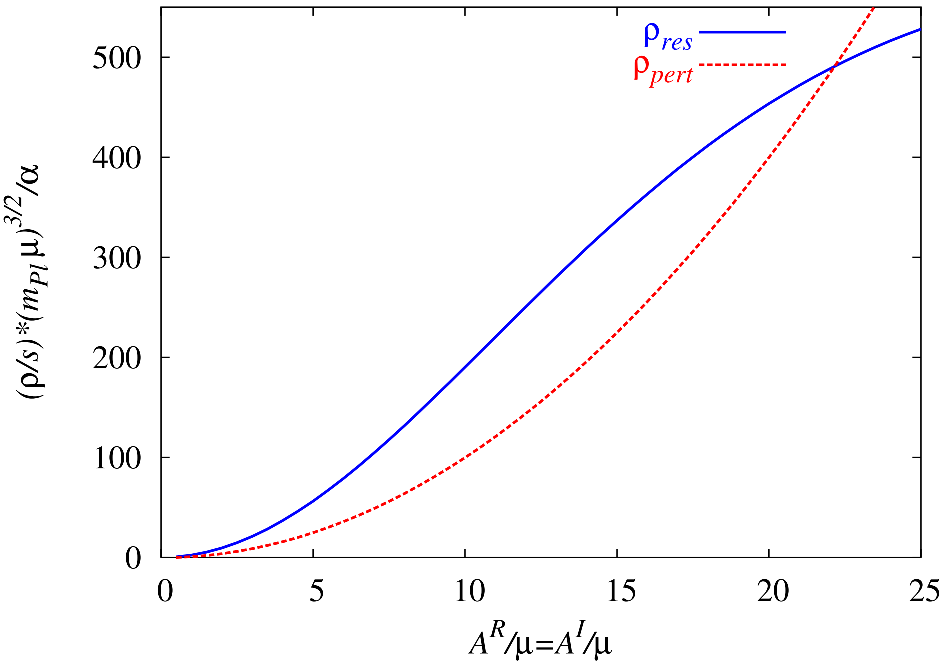

FIG. 3: The axial asymmetry plotted over the initial amplitude of $\phi_u^0 = A^R + \mathrm{i}A^I$. The choice of parameters is $\lambda_\nu = 0.2$, $m = 0.05\mu$, $\Gamma = 0.002\mu$.

4. CONCLUSIONS

Nonperturbatively sourced single field baryogenesis is a viable scenario. When compared to the perturbative source, it gets enhanced by a larger coupling λ_ν of the scalar condensate to the neutrinos and by a smaller decay rate Γ. Note however, that since the rate for the process $\phi_u \to \bar{\nu} + \nu$ is $\Gamma_\nu = \lambda_\nu^2 \mu/(8\pi) \approx 1.6 \times 10^{-3}\mu$, both numbers are not independent, and the second example we present is close to the case where the only channel of decay for ϕ_u is into neutrinos. For the numerical examples presented in this paper we consistently choose damping rates $\Gamma > \Gamma_\nu$. Processes of nonperturbative particle production may therefore be of importance for explaining the baryon asymmetry of the Universe. Besides the model presented here, the coherent baryogenesis mechanism is an example.

Acknowledgments

We thank R. H. Brandenberger and K. Kainulainen for participation in early stages of this work and A. Notari for useful discussions. The work of KB is funded by NSERC (Canada) and by the Fonds de Recherche sur la Nature et les Technologies du Québec.

---

[1] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5**, 32 (1967) [JETP Lett. **5**, 24 (1967 SOPUA,34,392-393.1991 UFNAA,161,61-64.1991)]; For a review on baryogenesis, see A. Riotto and M. Trodden, Ann. Rev. Nucl. Part. Sci. **49**, 35 (1999) [arXiv:hep-ph/9901362].

[2] Super–Kamiokande Collaboration, Y. Fukuda, Phys. Rev. Lett. **81**, 1562 (1998); Phys. Lett. **B467**, 185 (1999); GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. **B447**, 127 (1999); SAGE Collaboration, J. N. Abdurashitov *et al.* Phys. Rev. **C60**, 055801 (1999); Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996); Homestake Collaboration, B.T. Cleveland *et al.*, Astrophys. J. **496** 505 (1998); GNO Collaboration, M. Altmann *et al.*, Phys. Lett. **B490** 16 (2000); SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **87**, 071301 (2001): S. Fukuda *et al.,* Super–Kamiokande Collaboration, Phys.Lett. **B539**, 197 (2002); S.N. Ahmed *et al.,* SNO Collaboration, arXiv:nucl-ex/0309004; Super–Kamiokande Collaboration, Y. Ashie *et al.*, hep-ex/0404034.

[3] M. Fukugita, T. Yanagida, Phys.Lett. **B174**, 45, (1986); for reviews, see, e.g. W. Buchmuller, R. D. Peccei and T. Yanagida, arXiv:hep-ph/0502169; W. Buchmüller, M. Plümacher, hep-ph/0007176; A. Pilaftis, Int. J. Mod. Phys. **A14**; 1811 (1999).

[4] K. Dick, M. Lindner, M. Ratz and D. Wright, Phys. Rev. Lett. **84** (2000) 4039 [arXiv:hep-ph/9907562].

[5] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985) 36.

[6] T. Konstandin, T. Prokopec, M. G. Schmidt and M. Seco, arXiv:hep-ph/0505103.

[7] K. R. S. Balaji and R. H. Brandenberger, Phys. Rev. Lett. **94** (2005) 031301 [arXiv:hep-ph/0407090]; K. R. S. Balaji, R. H. Brandenberger and A. Notari, arXiv:hep-ph/0412197.

[8] I. Affleck and M. Dine, Nucl. Phys. B **249** (1985) 361.

[9] J. H. Traschen and R. H. Brandenberger, Phys. Rev. D **42** (1990) 2491.

[10] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D **56** (1997) 3258 [arXiv:hep-ph/9704452]; D. J. H. Chung, E. W. Kolb, A. Riotto and I. I. Tkachev, Phys. Rev. D **62** (2000) 043508 [arXiv:hep-ph/9910437]; G. F. Giudice, M. Peloso, A. Riotto and I. Tkachev, JHEP **9908** (1999) 014 [arXiv:hep-ph/9905242].

[11] M. Postma and A. Mazumdar, JCAP **0401** (2004) 005 [arXiv:hep-ph/0304246].

[12] B. Garbrecht, T. Prokopec and M. G. Schmidt, Phys. Rev. Lett. **92** (2004) 061303 [arXiv:hep-ph/0304088].

[13] B. Garbrecht, T. Prokopec and M. G. Schmidt, Eur. Phys. J. C **38** (2004) 135 [arXiv:hep-th/0211219].