LARGE SCALE LINAC SIMULATIONS USING A GLOBALISED SCATTERING MATRIX APPROACH

I. Shinton and R.M. Jones; Cockcroft Institute, Daresbury; and The University of Manchester, UK

Abstract

A globalised cascaded scattering matrix scheme serves as practical method to simulate the electromagnetic (e.m.) fields in the groups of cavities which constitute the main accelerating structures of a linac. The cascaded scattering matrix technique allows realistic fabrication errors to be incorporated in an efficient manner without the necessity to re-mesh the entire geometry. Once the unit cell structures have been determined using a numerical scheme, such as finite element method utilized here, the overall cascaded scattering matrix calculation requires little in the way of computational resources or time and is consequently an efficient means of characterizing the e.m. field. Details of the e.m. field and trapped modes for large scale linac simulations applied to the baseline cavities for the ILC and applications to XFEL are presented.

INTRODUCTION

In large accelerating structures such as the ILC (in both its baseline and alternative higher gradient designs) beam break up and emmitance dilution are major design concerns; hence the need to be able to accurately model large fractions of these structures in which effects such as wakefields, trapped modes, coupler kicks have been taken into consideration.

The modelling procedure is further complicated by machining and alignment errors. The effects of these errors needs to be ascertained, particularly for the transverse higher order modes (HOMs). A full scale simulation of large sections of the ILC is beyond what can presently be achieved by the traditionally employed numerical methods (such as the finite element method). Moreover the inclusion of realistic defects for a root mean squared (RMS) calculation of many machines will be prohibitively time consuming as it will require re-meshing of the problem domain.

The generalised scattering matrix technique is a mature RF method [1] requiring little in the way of computational resources or time allowing large structures, beyond the means of the traditionally employed numerical techniques, to be modelled. The generalised scattering matrix technique (and similar methods such as the coupled scattering calculation CSC [2], [3], [4]) has been shown to be capable of rapidly and accurately simulating structures [5]. The technique is very accurate as demonstrated in [5] and is capable of incorporating misalignments and defects into the calculation in an efficient manner (refer to [6]) allowing rapid RMS calculations to be preformed.

In practice it is necessary to obtain the electromagnetic field for a structure in its entirety. This aspect using GSM (or a similar scattering matrix technique) is discussed in the following section.

Presented in this paper is a new GSM formulation which allows rapid field calculation across an accelerating structure and the location of potentially trapped modes.

S MATRIX FORMULATIONS

There are broadly speaking three main methods, or subcategories, by which the e.m. field may be re-derived from S matrices: 1) performing an eigen mode simulation in which a planar wave is applied at each of the subsections (this is the method employed by CSC); 2) considering the absolute value of the wave amplitudes of the electrical fields at the ports of the subsections; and 3) mode matching.

CSC is a method whereby individual S matrices of various subsections of a large structure are combined and used to determine the amplitudes at various ports of the subsections [2], [3], [4]. This method has the ability to determine the scattering properties of irregular structures (including cavities with couplers). The electromagnetic fields using CSC are determined using the amplitudes at the individual ports of each subsection as boundary conditions in a separate eigen mode calculation in which a planar wave with the calculated amplitude is excited at each subsection port [2], [3], [4]. This method of determining the e.m. field may also be applied to the GSM technique, in which a recursive S matrix tracking scheme may be used [7], [8] to determine the amplitudes at each of the subsections which are then used as boundary conditions in separate calculations [7]. The disadvantage is that one would have to perform many separate calculations to re-derive the electromagnetic field.

A practical and efficient means of looking at the field distribution within a large accelerating structure is to focus on the field at a series of points within the structure. This is the idea that was utilised in a simplified version of CSC [9] in which the absolute value of the wave amplitudes at the ports of the subsections was calculated. This allows a rapid picture of the field distribution at the ports of the subsections to be ascertained and permits trapped modes to be quickly determined.

There are a number of mode matching schemes used to re-derive the e.m. field from a GSM calculation [8], [10], [11], [12]. The implementation of the previous mode matching schemes to accelerator structures are based upon calculating the amplitudes of the excited modes via a beam current. Using an orthogonality condition at an interface between regions where the field must be continuous allows the field to be calculated within the surrounding transitional sections.

PHYSICAL INTERPRETATION OF A GSM TECHNQIUE FOR RAPID FIELD DETERMINATION

The model described below is based on the following assumptions: 1) the structure is a travelling wave structure and 2) the travelling waves are fed into the structure via one region/port and are then considered to propagate in that direction from region I to II. Consider a transition between two regions as depicted in Fig. 1.

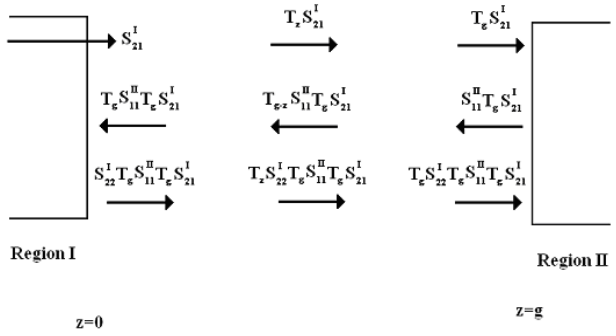

Figure 1: Representation of the series generated from a wave infinitely reflected within a transition.

An incoming wave from region I i.e. S_{21}^{I} will propagate across the transition decaying according to $T_z = \delta_{nm} e^{-jk_n z}$ (in which the subscripts n and m relate to the mode numbers and δ is a Kronecker function i.e. $\delta_{n=m} = 1, \delta_{n\neq m} = 0$). When it reaches region II it will be reflected back to region I and will gain S_{11}^{II} at the boundary $z = T_g$. The wave travels back decaying according to $T_{g-z} = \delta_{nm} e^{-jk_n(g-z)}$ until it reaches region I where it is reflected back to region II and gains S_{22}^{I}. This process is repeated an infinite number of times and may be written as the form of Eq. (1), where U is the Identity matrix. In a similar fashion the reflected waves propagating from region II can be written in the form of Eq. (2) in which the reflected wave $S_{11}^{\mathrm{II}} T_g S_{21}^{\mathrm{I}}$ travels back to region I decaying according to $T_{g-z} = \delta_{nm} e^{-jk_n(g-z)}$; upon each successive reflection the reflected wave acquires an additional multiple of $S_{11}^{\mathrm{II}} T_g S_{22}^{\mathrm{I}} T_g$. Note in Eq. (2) $T_{-z} = \delta_{nm} e^{jk_n z}$.

$$S_{21}^{0} = T_z \sum_{n=0}^{\infty} \left(S_{22}^{\mathrm{I}} T_g S_{11}^{\mathrm{II}} T_g \right)^n S_{21}^{\mathrm{I}} = T_z \left[U - S_{22}^{\mathrm{I}} T_g S_{11}^{\mathrm{II}} T_g \right]^{-1} S_{21}^{\mathrm{I}} \tag{1}$$

$$S_{11}^{0} = T_{-z} \left[U + \sum_{n=1}^{\infty} \left(T_g S_{11}^{\mathrm{II}} T_g S_{22}^{\mathrm{I}} \right)^n \right] T_g S_{11}^{\mathrm{II}} T_g S_{21}^{\mathrm{I}} = T_{-z} \left[U - T_g S_{11}^{\mathrm{II}} T_g S_{22}^{\mathrm{I}} \right]^{-1} T_g S_{11}^{\mathrm{II}} T_g S_{21}^{\mathrm{I}} \tag{2}$$

Utilising the transverse mode matching procedure we can describe the electric field components (transverse $\perp$ and longitudinal $\|$) in terms of Eq. (3) and (4).

$$\mathrm{E}_{\perp} = \sum_{1}^{N} \left(T_z S_{21}^{0} + T_{-z} S_{11}^{0} \right) \vec{e}_n \tag{3}$$

$$\mathrm{E}_{\|} = \sum_{1}^{N} \left(T_z S_{21}^{0} - T_{-z} S_{11}^{0} \right) Y_n \, \vec{e}_n \tag{4}$$

In Eq. (3) and (4) $\vec{e}_n$ is the field pattern of mode "n" [13] in which Y is the admittance. If we consider an infinitely small gap length between region I and II i.e. $T_z = T_{-z} = \delta_{nm} = U$ then we obtain Eq. (5) and (6).

$$\mathrm{E}_{\perp} = \sum_{1}^{N} \left(S_{21}^{0} + S_{11}^{0} \right) \vec{e}_n \tag{5}$$

$$\mathrm{E}_{\|} = \sum_{1}^{N} \left(S_{21}^{0} - S_{11}^{0} \right) Y_n \, \vec{e}_n \tag{6}$$

Eq. (5) and (6) gives us the electric field across the ports used in a GSM technique, where S^{II} is the S matrix of the section cascaded from the right of the transition and S^{I} is the S matrix of the section cascaded from the left of the transition. S_{11}^{0} and S_{21}^{0} may be thought of as the wave amplitudes in equations 3 and 4. Eq. (1), (2), (5) and (6) are similar to the methodology employed by CSC [9] which allows one to quickly ascertain the fields across a structure at the transitional sections used to generate the GSM calculation.

As an example of this GSM field determination technique, let us consider a 9 cell TESLA structure operating in the 5$^{\mathrm{th}}$ dipole region [14] since this is a region where modes are potentially trapped. Comparison between the HFSSv11 simulations for a complete structure and those obtained using GSM are presented in Fig. 2 and 3, in which the field profile calculated using the GSM method has been normalised with respect to the maximum field value. In Fig. 2 an iris to iris unit cell structure was chosen for a 9 cell TESLA structure composed solely of middle cells. In Fig. 3 a more detailed picture of the field is generated using iris to equator unit cell structures for a complete TESLA 9 cell structure with beam pipes.

The unit cell structures required for the cascading calculations were determined using HFSSv11 where a driven modal solution was sought in which the FEM mesh was adaptively refined until an overall accuracy better than 0.01% was obtained for the resulting S parameters followed by a frequency sweep in linear steps of 1MHz. The dipole modes for the symmetrical structures calculated in this paper were modelled using a quarter of the geometry and E and H symmetry planes

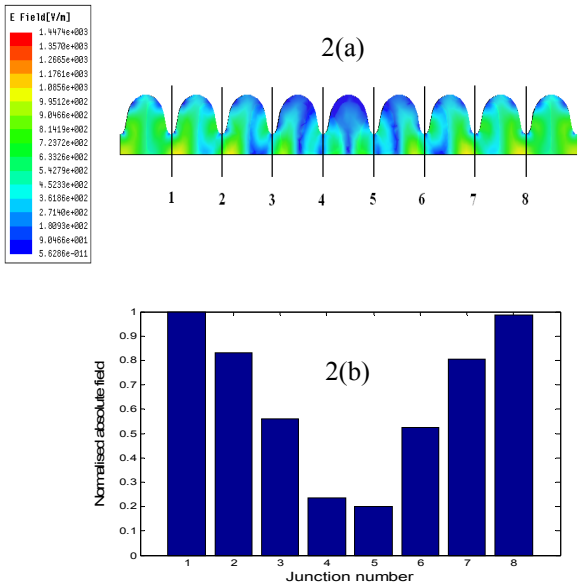

Figure 2: Field simulations of a 9 cell TESLA structure made up entirely of middle cells operating in the 5th dipole band at 3.039GHz. A wave is launched from the left region to excite the field. Displayed above is the first mode, the TE_{11} contribution of the total field. Above in 2(a) the full HFSS v11 simulation, below in 2(b) is the GSM field calculation.

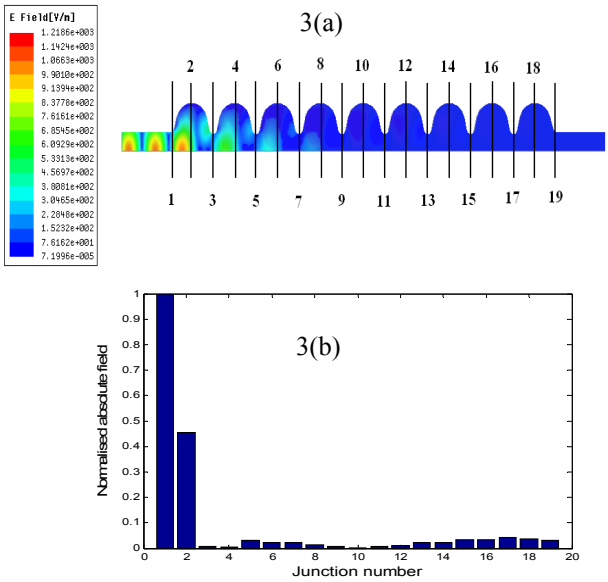

Figure 3: Field simulations of a full 9 cell TESLA structure operating in the 5th dipole band at 3.075GHz. A wave is launched from the left region to excite the field. Displayed above is the first mode, the TE_{11} contribution of the total field. Above in 3(a) is the full HFSS v11 simulation, below in 3(b) is the GSM field calculation.

DISCUSSION

The method presented here was derived for a travelling wave structure. A standing wave structure can be model by modifying the infinite series in Eq. (1) and (2) with the addition of the infinite series generated from an outgoing wave from region II (S_{11}^{II}). The main advantage of the method is that it allows rapid field determination across a structure as a function of frequency and position. Care must be taken to avoid missing trapped modes that may be overlooked as a consequence of the unit cell choice; if the field does not lie upon the chosen subsection point then a misleading representation of the field across the structure will be obtained. One way to circumvent this is to consider the overall geometry to be composed of many unit cells and in this way a more appropriate representation of the field distribution will be obtained (as seen in Fig. 3).

For the TESLA (or any other geometry which varies transversely) the propagation constant varies along the longitudinal axis for any given region. Previous mode matching methods have relied upon narrow wide narrow transitions (NWN) in which the propagation constant is a constant within a particular region. To circumvent this problem one could decompose the geometry into a series of NWN transitions. A practical calculation would therefore involve a sufficiently large proportion of NWN transitions of the geometry to adequately represent the field within the structure. An alternative approach to the other mode matching schemes previously mentioned would be to calculate the amplitudes of a transition as derived in the previous section using S_{11}^{0} and S_{21}^{0}. The development of this technique is the basis of ongoing work.

ACKNOWLEDGEMENTS

We have benefited from discussions at the weekly Manchester electrodynamics and wakefields meeting held at the Cockcroft Institute.

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